NATIONAL CERTIFICATE (VOCATIONAL)

SUBJECT GUIDELINES

MATHEMATICS
Level 4

IMPLEMENTATION: JANUARY 2013
INTRODUCTION

A. What is Mathematics?

*Reader's Digest Oxford Complete Word finder* defines Mathematics as “the abstract science of number, quantity and space studied in its own right”.

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. Through mathematical problem solving, students can understand the world and use that understanding in their daily lives.

Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. The Subject Outcomes and Assessment Standards for Mathematics are designed to allow all students to become citizens who will be able to confidently deal with Mathematics as and when it affects their daily lives, their community and the world in general.

B. Why is Mathematics important as a Fundamental subject?

The subject Mathematics (NQF Level 2 – 4) empowers students to:

- Communicate appropriately using descriptions in words, graphs, symbols, tables and diagrams.
- Use mathematical process skills to identify, pose and solve problems creatively and critically.
- Organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns.
- Work collaboratively in teams and groups to enhance mathematical understanding.
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions.
- Engage responsibly with quantitative arguments relating to local, national and global issues.

C. How do the Learning Outcomes link with the Critical and Developmental Outcomes?

The Learning Outcomes provide a platform for students to achieve the following Critical Cross field Outcomes and Developmental Outcomes:

- Identify and solve problems and make decisions using critical and creative thinking.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.
- Reflect on and explore a variety of strategies to learn more effectively.

D. Which factors contribute to achieving the Learning Outcomes?

A learning enabling environment for Mathematics is created by:

- Encouraging an attitude of “*I can do Mathematics*” in students.
- Using different media and learning approaches to accommodate different learning styles.
- Applying different strategies to develop and encourage creativity and problem solving capabilities.
- Focusing on strategies that develop higher level cognitive skills such as analytical and logical thinking and reasoning.
- Adopting a learning pace that will instil a sense of achievement rather than one of constant failure.
- Practical and relevant examples so that students can apply abstract concepts in real everyday life situations.
- Providing remedial and support interventions for those students that struggle to grasp fundamental outcomes.
- Encouraging continuous work and exercise for students to develop a sense of achievement and success.
MATHEMATICS – LEVEL 4

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1. DURATION AND TUITION TIME
This is a one year instructional programme comprising 200 teaching and learning hours. The subject may be offered on a part-time basis provided all of the assessment requirements set out hereunder are adhered to.

Students with special education needs (LSEN) must be catered for in a way that eliminates barriers to learning.

2. SUBJECT LEVEL OUTCOMES

SAQA Qualification ID: 50441

Students will be able to:

- Perform advanced operations on complex numbers and solve problems using complex numbers.
- Investigate and represent a wide range of algebraic expressions and functions and solve related problems.
- Use the Cartesian co-ordinate system to derive and apply equations.
- Explore, interpret and justify geometric relationships
- Solve problems by constructing and interpreting trigonometric models
- Analyse and interpret data to establish statistical models to solve related problems.
- Use experiments, simulation and probability distribution to set and explore probability models.
- Use mathematics to plan and control financial instruments.

3. ASSESSMENT

Information provided in this document on internal and external assessment aims to inform, assist and guide a lecturer to effectively plan the teaching of the subject.

The Assessment Guidelines for Mathematics Level 4, which compliments this document, provides detailed information to plan and conduct internal and external assessments and suggested mark allocations.

3.1 Internal assessment (25 percent)

Detailed information regarding internal assessment and moderation is outlined in the current ICASS Guideline document provided by the DHET

<table>
<thead>
<tr>
<th>Distribution of internal assessment components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three formal written tests &amp; one internal examination</td>
</tr>
<tr>
<td>Three assignments and one practical task/project</td>
</tr>
</tbody>
</table>

Possible spread of internal assessments during the year

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2-3</td>
<td>*2-3</td>
<td>0-1</td>
<td>7</td>
</tr>
</tbody>
</table>

*One of these must be an internal examination.

3.2 External assessment (75 percent)

A national examination is conducted annually in October/November by means of a paper/s set and moderated externally.

4. WEIGHTED VALUES OF THE TOPICS

<table>
<thead>
<tr>
<th>TOPICS</th>
<th>WEIGHTED VALUE</th>
<th>*TEACHING HOURS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Complex Numbers</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2. Functions and Algebra</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>3. Space, Shape and Measurement</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>4. Data Handling and Probability Models</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>5. Finance</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td>110</td>
</tr>
</tbody>
</table>

*Teaching Hours* refer to the minimum hours required for face to face instruction and teaching. This number excludes time spent on revision, test series and internal and external examination/assessment. The number of the allocated teaching hours is influenced by the topic weighting, complexity of the subject content and the duration of the academic year.

5. CALCULATION OF FINAL MARK

Continuous assessment: Student’s mark/100 x 25/1 = a mark out of 25 (a)
Examination mark: Student’s mark/100 x 75/1= a mark out of 75 (b)
Final mark: (a) + (b) = a mark out of 100

All marks are systematically processed and accurately recorded to be available as hard copy evidence for, amongst others, purposes of moderation and verification.

6. PASS REQUIREMENTS

The student must obtain minimum of 30 percent to pass the subject. A pass will be condoned at 25 percent if it is the only subject preventing the student from obtaining a level 4 certificate.

7. SUBJECT AND LEARNING OUTCOMES

On completion of Mathematics Level 4, the student should have covered the following topics:

- Topic 1: Complex numbers
- Topic 2: Functions and Algebra
- Topic 3: Space, Shape and Measurement
- Topic 4: Data Handling and Probability Models
- Topic 5: Financial Mathematics

**Topic 1: Complex Numbers**

(Minimum of 10 hours face to face teaching which excludes time for revision, test series and internal and external examination)

**Subject Outcome 1.1: Work with complex numbers.**

**Learning Outcome**

Students should be able to:
- Perform addition, subtraction, multiplication and division on complex numbers in standard form. (Includes i-notation)
  \[ \text{Note: Leave answers with positive argument} \]
- Perform multiplication and division on complex numbers in polar form.
- Use De Moivre’s theorem to raise complex numbers to powers (excluding fractional powers)
- Convert the form of complex numbers where needed to enable performance of advanced operations on complex numbers (a combination of standard and polar form may be assessed in one expression)

**Subject Outcome 1.2: Solve problems using complex numbers.**
Learning Outcomes
Students should be able to:
- Solve identical complex numbers in rectangular/standard form using the concept of simultaneous equations.
- Use complex numbers to solve equations that cannot be solved using the real number system by applying:
  - Factorisation
  - Quadratic formula

Topic 2: Functions and Algebra.
(Minimum of 40 hours face to face teaching which excludes time for revision, test series and internal and external examination)

Subject Outcome 2.1: Work with algebraic expressions using the remainder and the factor theorems.
Learning Outcomes:
Students should be able to:
- Use and apply the remainder and the factor theorem.
  - Find the remainder
  - Prove that an expression is a factor
  - Find an unknown variable in order to make an expression a factor or to leave a remainder.
- Factorise third degree polynomials including examples that require the factor theorem.
  (Long division or any other method may be used)

Subject Outcome 2.2: Use a variety of techniques to sketch and interpret graphs of the inverse of a function.
Learning Outcomes
Students should be able to:
- Determine the equations of the inverses of the functions:
  \[ y = ax + q \]
  \[ y = ax^2 \]
  \[ y = a^x ; a > 0 \]
  (\( y = a^x \) may be left with \( x \) as the subject of the formula. Note: No logarithms required)
- Sketch the graphs of the inverse of the functions:
  \[ y = ax + q \]
  \[ y = ax^2 \]
  \[ y = a^x ; a > 0 \]
  Note: Sketching the graphs using point by point plotting is an option

Obtain the equation of any of the following inverse graphs given as a sketch.
\[ y = ax + q \]
\[ y = ax^2 \]
\[ y = a^x ; a > 0 \]
• Identify characteristics as listed below with respect to the following functions and their inverses.

\[ y = ax + q \]
\[ y = ax^2 \]
\[ y = a^x ; a > 0 \]

- Domain and range.
- Intercepts with axes.
- Turning points, minima and maxima.
- Asymptotes
- Shape and symmetry.
- Functions or non-functions.
- Continuous or discontinuous.
- Intervals at which a function increases/decreases.

Subject Outcomes 2.3: Use mathematical models to investigate linear programming problems.

Learning Outcome
Students should be able to:
- Find and formulate the linear constraints from a given problem.
- Solve linear programming problems by optimizing a function in two variables, subject to one or more linear constraints, using the search line method.

Method:
- Sketch the functions/constraints.
- Determine and shade the feasible region.
- Work out the gradient of the search line.
- Use the search line to optimise the maximum or minimum from the objective function.

Subject Outcomes 2.4: Investigate and use instantaneous rate of change of a variable when interpreting models both in mathematical and real life situations.

Learning Outcomes
Students should be able to:
- Establish the derivatives of the following functions from first principles:
  \[ f(x) = b \]
  \[ f(x) = ax + b \]
  \[ f(x) = ax^2 + b \]
  \[ f(x) = x^3 \]
  \[ f(x) = ax^3 \]
  \[ f(x) = \frac{1}{x} \]
  \[ f(x) = \frac{a}{x} \]

Note: The binomial theorem does not form part of the curriculum.

- Find the derivatives of the functions in the form:
\[ f(x) = ax^n \]
\[ f(x) = a \ln kx \]
\[ f(x) = ae^{kx} \]
\[ f(x) = a \sin kx \]
\[ f(x) = a \cos kx \]
\[ f(x) = a \tan kx \]

Where

\[ f(x) = ax^n \quad f'(x) = n \cdot ax^{n-1} \]
\[ f(x) = \ln kx \quad f'(x) = \frac{k}{x} \]
\[ f(x) = e^{kx} \quad f'(x) = ke^{kx} \]
\[ f(x) = a \sin kx \quad f'(x) = ka \cos kx \]
\[ f(x) = a \cos kx \quad f'(x) = -ka \sin kx \]

Examples to include are

\[ 3x^2 \quad \frac{3}{x^3} \quad \frac{-2}{\sqrt{x^2}} \quad 2 \ln 3x \quad \frac{1}{2} e^{-2x} \]
\[ 2 \sin 3x \quad \frac{1}{3} \cos \frac{x}{2} \quad -4 \tan x \quad \text{etc.} \]

- Use the constant, sum and/or difference, product, quotient and chain rules for differentiation.

*Note: Combinations of rules in the same problem are excluded.*

- Find the equation of the tangent to a graph at a specific point.

- Solve practical problems involving rates of change.

*Note: velocity and acceleration may be included*

- Draw graphs of cubic functions by determining:
  - y-intercept
  - roots (x-intercepts)
  - turning points using derivatives

- Determine/prove maximum and minimum turning points by making use of second order derivatives
  (Only: quadratic and cubic functions)

**Subject Outcome 2.5:** Analyse and represent mathematical and contextual situations using integrals and find areas under curves by using integration rules.

**Learning Outcomes**

Students should be able to:

- Find the integrals of the following:
\[ \int ax^n \, dx \]
\[ \int \frac{a}{x} \, dx \]
\[ \int ae^{kx} \, dx \]
\[ \int a \sin kx \, dx \]
\[ \int a \cos kx \, dx \]
\[ \int a \sec^2 kx \, dx \]

Where:
\[ \int ax^n \, dx = \frac{a x^{n+1}}{n+1} + c \]
\[ \int \frac{a}{x} \, dx = a \ln x + c \]
\[ \int ae^{kx} \, dx = \frac{a e^{kx}}{k} + c \]
\[ \int a \sin kx \, dx = -\frac{a \cos kx}{k} + c \]
\[ \int a \cos kx \, dx = \frac{a \sin kx}{k} + c \]

Note:
- Simplifications may be required where necessary
- Integrals of polynomials may be assessed
- Integration by parts is excluded

- Use the upper and lower limits to calculate definite integrals.
- Determine the area under a curve by:
  - Working from a given graph or by sketching a graph
  - Working with an area bounded by a curve, the \( x \)-axis, an upper and a lower limit.
  - Splitting the area into two intervals when the graph crosses the \( x \)-axis

  **Note:**
  - Integrals with respect to the \( x \)-axis only.
  - Areas between two curves are excluded.
  - The \( y \)-axis (\( x = 0 \)) may be used as an upper or lower limit.

**Topic 3: Space, Shape and Measurement**

(Minimum of 35 hours face to face teaching which excludes time for revision, test series and internal and external examination)

**Subject Outcome 3.1: Use the Cartesian co-ordinate system to derive and apply equations.**

**Learning Outcomes**

Students should be able to:
- Use the Cartesian co-ordinate system to derive and apply the equation of a circle (any centre).
- Use the Cartesian co-ordinate system to derive and apply the equation of a tangent to a circle given a point on the circle.
Note:
- Straight lines to be written in the following forms only:
  \[ y = mx + c \quad \text{or} \quad y - y_1 = m(x - x_1) \]
  \[ \text{and/or} \quad ax + by + c = 0 \ (\text{general form}) \]

- Learners are expected to know and be able to use as an axiom “the tangent to a circle is perpendicular to the radius drawn to the point of contact.”

Subject Outcome 3.2: Explore, interpret and justify geometric relationships.

Learning Outcomes
Students should be able to:
- Use geometry of straight lines and triangles to solve problems and to justify relationships in geometric figures.
  Concepts to include are:
  - angles of a triangle;
  - exterior angles,
  - straight lines,
  - vertically opposite angles;
  - corresponding angles,
  - co-interior angles and
  - alternate angles.
- State and apply the following theorems of circles:
  - If a line is drawn from the centre of a circle to the midpoint of a chord, then that line is perpendicular to the chord.
  - If a line is drawn from the centre of the circle perpendicular to the chord, then it bisects the chord.
  - If an arc subtends an angle at the centre of the circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference.
  - If the diameter of a circle subtends an angle at the circumference, then the angle subtended is a right angle triangle.
  - If an angle subtended by a chord at a point on the circumference is a right angle, then the chord is a diameter.
  - Angles in the same segment of a circle are equal.
  - The opposite angles of a cyclic quadrilateral are supplementary.
  - If the exterior angle of a quadrilateral is equal to the interior opposite angle the quadrilateral will be a cyclic quadrilateral.
  - The four vertices of a quadrilateral in which the opposite angles are supplementary will be a cyclic quadrilateral.
  - If a tangent to a circle is drawn, then it is perpendicular to the radius at the point of contact.
  - If a line is drawn perpendicular to a radius at the point where the radius meets the circle, then it is a tangent to the circle.
  - If two tangents are drawn from the same point outside a circle then they are equal in length.
  - The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.(tan-chord theorem)

Note: Proofs of the above theorems are excluded

Subject Outcome 3.3: Solve problems by constructing and interpreting trigonometric models.

Learning Outcomes
Students should be able to:
- Use the following compound angle identities:
\[
\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta
\]
\[
\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta
\]

to derive and apply the following double angle identities:

\[
\sin 2\alpha = 2 \sin \alpha \cos \alpha
\]

\[
\cos 2\alpha = \begin{cases}
\cos^2 \alpha - \sin^2 \alpha \\
2 \cos^2 \alpha - 1 \\
1 - 2 \sin^2 \alpha
\end{cases}
\]

- Determine the specific solutions of trigonometric expressions using compound and double angle identities without a calculator.
  (e.g. \( \sin 120^\circ \), \( \cos 75^\circ \) etc.)
- Use compound angle identities to simplify trigonometric expressions and to prove trigonometric equations.
- Determine the specific solutions of trigonometric equations by using knowledge of compound angles and identities.

**Note:**
- **Solutions:** \([0;360^\circ]\)
- **Identities limited to:**
  \[
  \tan \theta = \frac{\sin \theta}{\cos \theta}
  \]
  \[
  \sin^2 \theta + \cos^2 \theta = 1
  \]
- Double and compound angle identities are included.
  **Note:** **radians are excluded**
- Solve problems from a given diagram in two and three dimensions by applying the sine and cosine rule.
  **Note:** **Area formula and compound angle identities are excluded.**

**Topic 4: Data Handling and Probability Models**

(Minimum of 18 hours face to face teaching which excludes time for revision, test series and internal and external examination)

**Subject Outcome 4.1: Represent, analyse and interpret data using various techniques.**

**Learning Outcomes**

Students should be able to:
- Identify situations or issues that can be dealt with through statistical methods.
  **Range:** Data given should include problems relating to health, social, economic, cultural, political and environmental issues.
  **Note:** Not for examination purposes but for class activities only.
- Discuss the use of appropriate and efficient methods to record, organise and interpret given data by making use of:
  - Manageable data sample sizes
    - (less than or equal to 10) and which are representative of the population.
- Graphical representations and numerical summaries which are consistent with the data, and clear and appropriate to the situation and target audience.
  - Note: Discussion only, not expected to draw again.
- Compare different representations of given data.

- Justify and apply statistics to answer questions about problems.
- Discuss new questions that arise from the modelling of data.
- Take a position on an issue by comparing different representations of given data.

**Subject Outcome 4.2: Use variance and regression analysis to interpolate and extrapolate bivariate data.**

**Learning Outcomes**

Students should be able to:

- Calculate:
  - variance and
  - standard deviation manually for small sets of data only.
- Interpret the meaning of variance and standard deviation for small sets of data only.
- Represent bivariate numerical data as a scatter plot
- Identify intuitively whether a linear, quadratic or exponential function would best fit the data.
- Draw the intuitive line of best fit.

**Range:**

- Data given should include problems related to health, social, economic, cultural, political and environmental issues.
- For small sets of data only (limited to 8)
- Use least squares regression method to determine a function which best fits a given set of bivariate data.
- Use the regression line to predict the outcome of a given problem

**Subject Outcome 4.3: Use experiments, simulation and probability distribution to set and explore probability models.**

**Learning Outcome**

Students should be able to:

- Explain and distinguish between the following terminology/events:
  - Probability
  - Dependent events
  - Independent events
  - Mutually exclusive
  - Mutually inclusive
  - Complimentary events
- Make predictions based on validated experimental or theoretical probabilities taking the following into account:
  - \( P(S) = 1 \) (where \( S \) is the sample space);
  - Disjoint (mutually exclusive) events, and is therefore able to calculate the probability of either of the events occurring by applying the addition rule for disjoint events:
- \( P(A \text{ or } B) = P(A) + P(B); \)
  - Complementary events and is therefore able to calculate the probability of an event not occurring;
- \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \) (where \( A \) and \( B \) are events within a sample space);
  - Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: \( P(A \text{ and } B) = P(A)P(B) \).

- Draw Tree diagrams, Venn diagrams and complete contingency two-way tables to solve probability problems (where events are not necessarily independent).

*Range:*
- Venn diagrams to be limited to two subsets.
- Tree diagrams where the sample space is manageable. (not more than 15 possible outcomes)
- Interpret and clearly communicate results of the experiments correctly in terms of real context

**Topic 5: Financial Mathematics**

(Minimum of 7 hours face to face teaching which excludes time for revision, test series and internal and external examination)

**Subject Outcomes 5.1:** Use mathematics to plan and control financial instruments.

**Learning Outcomes**

Students should be able to:
- Use simple and compound growth formulae
  \[ A = P(1 + in) \text{ and } A = P(1 + i)^n \text{ and } A = P \left(1 + \frac{r}{100 \times m}\right)^{m \times n} \]
  to solve problems, including interest, hire-purchase and inflation.
- Understand, use and interpret tax tables.
- Use simple and compound decay formulae, \( A = P(1 - in) \) and \( A = P(1 - i)^n \), to solve problems (straight line depreciations and depreciation on a reducing balance).

8. **RESOURCE NEEDS FOR THE TEACHING OF MATHEMATICS - LEVEL 4**

**Physical resources**
- Soft cover spring file for portfolios
- Scientific calculators
- Graph paper
- Textbook and workbook
- Memory stick
- Computer and printing facilities
- Data Projector
- Applicable graphing software
- Geometric sets
- Chalk and chalkboards
- Overhead Projectors
- Current newspapers and information about financial packages from banks and investment companies.
- Internet access or access to a good library or resource centre.
- Materials to build or create models
Human resources

A lecturer must have a Bachelors degree or equivalent recognised qualification and appropriate teaching experience to teach Level 4 Mathematics.