NATIONAL CERTIFICATES (VOCATIONAL)

SUBJECT GUIDELINES

MATHEMATICS

NQF LEVEL 2

IMPLEMENTATION: JANUARY 2011
INTRODUCTION

A. What is Mathematics?

*Reader’s Digest Oxford Complete Wordfinder* defines Mathematics as *“the abstract science of number, quantity and space studied in its own right.”*

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. Through mathematical problem solving, students develop an understanding of the world and can use that understanding to great effect in their daily lives.

Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. The Subject Outcomes and Assessment Standards for Mathematics are designed to allow all students to become citizens who will be able to confidently deal with Mathematics as and when it impinges on their daily lives, their community and the world in general.

B. Why is Mathematics important as a Fundamental subject?

The subject Mathematics (NQF Level 2 – 4) empowers students to:

- Communicate appropriately using numbers, verbal descriptions, graphs, symbols, tables and diagrams.
- Use mathematical process skills to identify, pose and solve problems creatively and critically.
- Organise, interpret and manage mathematical information which demonstrates responsibility and sensitivity to personal and broader societal concerns.
- Work collaboratively in teams and groups to promote understanding in general.
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions.
- Engage responsibly with quantitative arguments relating to local, national and global concerns.

C. How do the Learning Outcomes link with the Critical and Developmental Outcomes?

The Learning Outcomes provide a platform for students to achieve the following Critical Cross field Outcomes and Developmental Outcomes:

- Identify and solve problems and make decisions using critical and creative thinking.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.
- Reflect on and explore a variety of strategies to learn more effectively.

D. Which factors contribute to achieving the Learning Outcomes?

A learning enabling environment for Mathematics is created by:

- Encouraging an attitude of “I can do Mathematics” in students.
- Using different media and learning approaches to accommodate different learning styles.
- Applying different strategies to develop and encourage creativity and problem solving capabilities.
- Focusing on strategies that develop higher level cognitive skills such as analytical and logical thinking and reasoning.
- Adopting a learning pace that will instil a sense of achievement rather than one of constant failure.
- Practical and relevant examples so that students can apply abstract concepts in real everyday life situations.
- Providing remedial and support interventions for those students that struggle to grasp fundamental outcomes.
- Encouraging continuous work and exercise for students to develop a sense of achievement and success.
MATHEMATICS – LEVEL 2

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1. DURATION AND TUITION TIME
This is a one year instructional programme comprising 200 teaching and learning hours. The subject may be offered on a part-time basis provided all the assessment requirements are adhered to.
Students with special education needs (LSEN) must be catered for in a way that eliminates barriers to learning.

2. SUBJECT LEVEL OUTCOMES AND FOCUS
Students will be able to:
- Recognise and work with numbers and their relationships to estimate, calculate and check solutions.
- Investigate and represent a wide range of algebraic expressions and functions and solve related problems.
- Describe, represent, analyse and explain properties of shape in two- and three-dimensional space with justification.
- Analyse data to establish statistical models to solve related problems.
- Plan personal finances in the context of income and expenditure, basic budgets and the impact of interest rates.

3. ASSESSMENT REQUIREMENTS
3.1. Internal assessment (25 percent)
All internal assessments must be finalised by a competent assessor. Refer to the Assessment Guideline for Mathematics Level 2 for specific mark allocation on internal assessment.

| Three formal written tests & one internal examination | 70% of ICASS |
| Two assignments & one practical assessment | 30% of ICASS |

Possible distribution of tests, practical assessments and internal examination

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
<th>Total</th>
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<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>*2</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

*One of these must be an internal examination

- Some examples of practical assessments include, but are not limited to:
  A. Practical exercise work and applications to contextual problems
  B. Presentations (lectures, demonstrations, group discussions and activities, practical work, observation, role-play, self activity, judging and evaluation)
  C. Use of aids
  D. Exhibitions
  E. Visits

- Evidence in practical assessments
All evidence pertaining to evaluation of practical work must be reflected in the students’ Portfolios of Evidence (PoE).

3.1.1 Processing of internal assessment mark for the year
A year mark out of 100 is calculated from marks obtained during the internal continuous assessment (ICASS).
3.1.2 Moderation of internal assessment mark
Internal assessment is subjected to both internal and external moderation procedures as set out in the National Examination Policy for FET College Programmes.

3.2 External assessment (75 percent)
A National Examination is conducted annually in October or November by means of a paper/s set and moderated externally.

External assessment details and procedures are set out in Assessment Guidelines: Mathematics (Level 2).

4. WEIGHTED VALUES OF TOPICS

<table>
<thead>
<tr>
<th>TOPICS</th>
<th>WEIGHTED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Numbers</td>
<td>15</td>
</tr>
<tr>
<td>2. Functions and Algebra</td>
<td>25</td>
</tr>
<tr>
<td>3. Space, Shape and Measurement</td>
<td>30</td>
</tr>
<tr>
<td>4. Data Handling</td>
<td>20</td>
</tr>
<tr>
<td>5. Financial Mathematics</td>
<td>10</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>100</strong></td>
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</tbody>
</table>

5. CALCULATION OF FINAL MARK

Continuous assessment: \( \frac{X}{100} \times \frac{25}{1} = \text{a mark out of 25} \) (a)
Examination mark: \( \frac{X}{100} \times \frac{75}{1} = \text{a mark out of 75} \) (b)
Final mark: \( (a) + (b) = \text{a mark out of 100} \)

All marks are systematically processed and accurately recorded to be available as hard copy evidence for, amongst others, moderation and verification purposes.

6. PASS REQUIREMENTS

The student must obtain a minimum of 30 percent in Mathematics. A pass will be condoned at 25 percent if it is the only subject stopping the student from progressing to Level 3.

7. SUBJECT AND LEARNING OUTCOMES

On completion of Mathematics Level 2, the student should have covered the following topics:

Topic 1: Numbers
Topic 2: Functions and Algebra
Topic 3: Space, Shape and Measurement
Topic 4: Data Handling
Topic 5: Financial Mathematics

**Topic 1: Numbers**

Subject Outcome 1.1: Use computational tools and strategies and make estimates and approximations.

Learning Outcomes

Students are able to:
- Use a scientific calculator correctly to solve expressions involving addition, subtraction, multiplication, division, squares, cubes, square roots and cube roots.
- Estimate and approximate physical quantities to solve problems in practical situations. Quantities include length, time, mass and temperature

Subject Outcome 1.2: Demonstrate an understanding of numbers, relationships among numbers and number systems and represent numbers in different ways.

Learning Outcomes
Students are able to:

- Identify rational and irrational numbers.
- Round off rational and irrational numbers to an appropriate degree of accuracy.
- Convert rational numbers between terminating and recurring decimals to the form \( \frac{a}{b} ; a, b \in \mathbb{Z} ; b \neq 0 \).
- Apply the following laws of exponents.

\[
\begin{align*}
 a^m \times a^n &= a^{m+n} & a^m + a^n &= a^{m-n} \\
 (a^m)^n &= a^{m \times n} & (ab)^m &= a^m b^m \\
 (a^m b^n)^p &= a^{mp} b^{np} & \left( \frac{a}{b} \right)^m &= \frac{a^m}{b^m} \\
 \left( \frac{a^m}{b^n} \right)^p &= \frac{a^{mp}}{b^{np}} & a^{-n} &= \frac{1}{a^n} \\
 \frac{1}{a^{-n}} &= a^n & a^0 &= 1 \\
 \sqrt[n]{a^m} &= a^{\frac{m}{n}}
\end{align*}
\]

- Rationalise fractions with surd denominators (binomial and monomial denominators) without using a calculator.
- Add, subtract, multiply and divide simple surds.
- Manipulate simple technical and non-technical formulae.
- Solve an unknown variable in simple technical and non-technical formulae.
- Identify and work with arithmetic sequences and series.

**Topic 2: Functions and Algebra.**

**Subject Outcome 2.1:** Use a variety of techniques to sketch and interpret information from graphs of algebraic and transcendental functions.

**Learning Outcomes**

Students are able to:

- Generate graphs by means of point-by-point plotting supported by available technology.
- Use the generated graphs to make and test conjectures.
- Generalise the effects of the parameters \( a \) and \( q \) on the generated graphs of functions including the following:

\[
\begin{align*}
 y &= ax + q \\
 y &= ax^2 + q \\
 y &= \frac{a}{x} + q \\
 y &= ab^x + q ; b > 0 \\
 y &= a \sin x + q \\
 y &= a \cos x + q \\
 y &= a \tan x + q
\end{align*}
\]

- Define functions
Identify the following characteristics with of functions:
- Domain and range.
- Intercepts with axes.
- Turning points, minima and maxima.
- Asymptotes
- Shape and symmetry.
- Periodicity and amplitude
- Functions or non functions.
- Continuous or discontinuous.
- Sketch graphs and find equations of graphs for the following functions:
  \[ y = ax + q \]
  \[ y = ax^2 + q \]
  \[ y = \frac{a}{x} + q \]
  \[ y = ab^x + q \], \( b > 0 \)
  \[ y = a \sin x + q \]
  \[ y = a \cos x + q \]
  \[ y = a \tan x + q \]

Subject Outcome 2.2: Manipulate and simplify algebraic expressions.

Learning Outcomes
Students are able to:
- Find products of two binomials
- Find products of binomials with trinomials
- Factorise by identifying/ taking out of common factor.
- Factorise by grouping in pairs
- Factorise the difference of two squares.
- Factorise trinomials.
- Simplify algebraic fractions with monomial denominators.

Subject Outcome 2.3: Solve algebraic equations and inequalities.

Learning Outcomes
Students are able to:
- Solve linear equations.
- Solve quadratic equations by factorization.
- Solve exponential equations in the form \( k a^x = m \) (where \( x \) is an integer) by using the laws of exponents.
- Solve inequalities in one variable and represent the solution in set builder notation, interval notation and on the number line.
- Solve simultaneous equations with two unknowns algebraically and graphically, where both equations are linear.

Topic 3: Space, Shape and Measurement.

Subject Outcome 3.1: Measure and calculate physical quantities.

Learning Outcomes
Students are able to:
- Read scales on measuring instruments correctly. Instruments to include the ruler and protractor.
- Use symbols and Systeme Internationale (SI) units as appropriate to the situation.

Subject Outcome 3.2: Calculate perimeter, surface area and volume in two and three dimensional geometrical shapes.

Learning Outcomes
Students are able to:

- Calculate the perimeter and surface area of the following laminas:
  - Square
  - Rectangle
  - Circle
  - Triangle
  - Parallelogram
  - Trapezium
  - Hexagons

- Calculate the volume of the following geometric objects:
  - Cubes
  - Rectangular prisms
  - Cylinders
  - Triangular prisms
  - Hexagonal prisms

- Investigate the effect on area of laminas where one or more dimensions are multiplied by a constant factor $k$ is investigated

- Investigate the effect on the volume and surface area of right prisms and cylinders, where one or more dimensions are multiplied by a constant factor $k$

Subject Outcome 3.3: Use the Cartesian co-ordinate system to derive and apply equations.

Learning Outcomes

Students are able to:

- Use the Cartesian co-ordinate system to plot points, lines and polygons.
- Use the Cartesian co-ordinate system to calculate the distance between two points.
- Use the Cartesian co-ordinate system to find the gradient of the line joining two points.
- Use the Cartesian co-ordinate system to find the co-ordinates of the midpoint of a line segment joining two points

Subject Outcome 3.4: Use and apply transformations to plot co-ordinates.

Learning Outcomes

Students are able to:

- Find the co-ordinates of the point $(x; y)$ after it is translated $p$ units horizontally and $q$ units vertically.
- Find the co-ordinates of the point $(x; y)$ after it is reflected about the $x$-axis, the $y$-axis, and the line $y = -x$ and the line $y = x$.

Subject Outcome 3.5: Solve problems by constructing and interpreting geometrical models.

Learning Outcomes

Students are able to:

- Investigate the relationship between the sides of a right-angled triangle to develop the Theorem of Pythagoras.
- Use the Theorem of Pythagoras to calculate a missing length in a right-angled triangle leaving answers in the most appropriate form.

Subject Outcome 3.6: Solve problems by constructing and interpreting trigonometric models.

Learning Outcomes

Students are able to:

- Define and use the following trigonometric functions: $\sin \theta$; $\cos \theta$; $\tan \theta$;
- Calculate trigonometric ratios in each of the quadrants where one ratio in that quadrant is given $\sin \theta = \frac{3}{5}$ and $90^\circ \leq \theta \leq 180^\circ$
• Solve problems in two dimensions by using the trigonometric functions \( \sin \theta \); \( \cos \theta \); \( \tan \theta \).
• Express an appreciation of the contribution to the history of the development and the use of geometry and trigonometry by various cultures.
(Not to be examined, delivered by means of a presentation by the lecturer or completed as a project)

**Topic 4: Data Handling**

**Subject Outcome 4.1:** Calculate central tendencies and dispersion of data.

**Learning Outcomes**

Students are able to:

- Calculate central tendency of ungrouped data, namely the mean, median and mode.
- Calculate measures of dispersion including range, percentiles, quartiles, inter-quartile and semi-inter-quartile range.

**Subject Outcome 4.2:** Represent data effectively.

**Learning Outcomes**

Students are able to:

- Represent data effectively, choosing appropriately from:
  - Construction of Frequency Distribution/Tally Chart
  - Bar and compound bar graphs;
  - Construction of the stem and leaf plot;
  - Histograms (grouped data);
  - Frequency polygons;
  - Pie charts;
  - Line and broken line graphs.

**Topic 5: Financial Mathematics**

**Subject Outcome 5.1:** Plan and manage personal and household finances.

**Learning Outcomes**

Students are able to:

- Describe financial concepts related to personal finances, methods of financing and financial control.

  *Range: Needs, wants, salary, wage, income, expense, budget, fixed expense, variable expense, savings account, cheque account, short term investment, medium term investment, fixed deposit, endowment, long term investment, credit card account, debit card, credit card, bank fees, variance, cheque book, unit trusts, retirement annuities, pension fund, interest, interest rate, simple interest, compound interest, principle amount, hire purchase agreement, stokvels, mashonisa*

- Draw up a projected personal and household monthly budget.
- Record actual income and expenditure over a period (one month, six months or twelve months) and compare to the projected budget.
- Identify and explain variances between actual and projected figures
- Provide possible corrective methods of financial control.

**Subject Outcome 5.2:** Use simple and compound interest to explain and define a variety of situations.

**Learning Outcomes**

Students are able to:

- Differentiate between simple and compound interest.
- Explain the advantages and disadvantages of using simple and compound interest in specific situations.
- Use and manipulate the simple growth formula \( A = P(1 + i n) \) to solve problems.
- Use and manipulate the compound growth formula \( A = P(1 + i)^n \) to solve problems subject to only annual compounding being made.

  *(Range: Manipulation of only \( A; P; i \ ))*
8. RESOURCE NEEDS FOR THE TEACHING OF MATHEMATICS – LEVEL 2

- **Physical resources**
  - Files for portfolios
  - Scientific calculators
  - Graph paper
  - Textbook or workbook
  - Computer stiffy or memory stick
  - Computer and printing facilities
  - Data Projector
  - Applicable graphing software
  - Geometric sets
  - Chalk and chalkboards
  - Paper
  - Overhead Projectors
  - Current newspapers and information about financial packages from banks and investment companies.
  - Internet access or access to a good library or resource centre.
  - Models

- **Human resources**
  
  A lecturer must have NQF Level 5 Mathematics or an equivalent with an appropriate teaching qualification to teach Level 2 Mathematics.